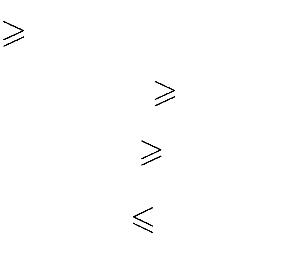
2017 年福建省普通高中毕业班单科质量检查



理科数学试题答案及评分参考

评分说明：

1．本解答给出了一种或几种解法供参考，如果考生的解法与本解答不同，可根据试题的主要考

查内容比照评分标准制定相应的评分细则。

2．对计算题，当考生的解答在某一步出现错误时，如果后继部分的解答未改变该题的内容和难

度，可视影响的程度决定后继部分的给分，但不得超过该部分正确解答应给分数的一半；如果后继部

分的解答有较严重的错误，就不再给分。

3．解答右端所注分数，表示考生正确做到这一步应得的累加分数。

4．只给整数分数。选择题和填空题不给中间分。

一、选择题：本大题考查基础知识和基本运算．每小题 5分，满分 60分．

（1）C （2）C （3）A （4）B （5）A （6）D

（7）C （8）D （9）C （10）B （11）B （12）D

二、填空题：本大题考查基础知识和基本运算．每小题 5分，满分 20分．

（13） 1 （14） 6 （15） *y* *x* （16） 21π

三、解答题：本大题共 6小题，共 70分．解答应写出文字说明、证明过程或演算步骤．

（17）本小题主要考查等差数列、等比数列、数列求和等基础知识，考查运算求解能力，考查函数与

方程思想、分类与整合思想、化归与转化思想等．满分 12 分．

解法一：（Ⅰ）因为 *S* 2*a* 1， ①

*n* *n*

所以当 *n* 1时，

*a*1  *S*1  2*a*1 1，解得 *a*1 1，··············································· 1 分

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 当 *n* 2时， | | |  | *S* 1  2*a* 1 1， ② ······························································ 2 分  *n* *n* |
|  | ①-②，得 |  | *a* 2*a* 2*a* ，即 2 ，所以 2*n*1  *a* *a* *a* .·································· 3 分  *n* *n* *n*1 *n*  *n* *n*1 | | | |

由数列

*b* 的前三项和为3，得 *b*2 1. ······································· 4 分

3*b* 3 ，所以

*n* 2

设数列*b* 的公差为 *d* ，则*b*3 1 *d*,*b*5 13*d* ，··········································· 5 分

*n*

又因为 2

*b* *b* *b* ，所以 (1 *d*)2 1 3*d* ，

3 2 5

解得 *d* 1或 *d* 0 （舍去），所以*b* *n* 1．·················································· 6 分

*n*

（Ⅱ）由（Ⅰ），可知 *a* 2*n*1 ，*b* *n* 1，从而 *a* *b* (*n* 1)2*n*1 ，

*n* *n* *n* *n*

令

*T* *a* *b* *a* *b* *a* *b* ，

*n* 1 1 2 2 *n* *n*

即*T* 121  222  (*n* 2)2*n*2  (*n* 1)2*n*1 ， ③ ······························· 7 分

*n*

③×2，得 *T* *n*  *n* ， ④··················· 8 分

2 1 2 2 2 ( 2) 2*n*1 2*n*

2 3 1

*n*

③-④，得 *T* *n*

············································· 9 分

2 22 23 2*n* 1 1 2*n*

*n*

|  |  |  |  |
| --- | --- | --- | --- |
|  | 2 2  *n*  *n* 1 2*n* *n* 2 2*n* 2    1 2 |  | ， |

即 2 2*n*2

*T* *n* . ···············································································10 分

*n*

故题设不等式可化为 (*n* 2)2*n*(*n* 2)*t* ， （\*）

（ⅰ）当 *n* 1时，不等式（\*）可化为 2 *t* ，解得 *t* 2 ；

（ⅱ）当 *n* 2 时，不等式（\*）可化为 0 0 ，此时*t* **R** ；

（ⅲ）当 *n* 3 时，不等式（\*）可化为*t* 2*n*，因为数列2 *n*是递增数列，所以*t* 8.

综上，*t* 的取值范围是2，8.····································································12 分

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解法二：（Ⅰ）因为 *S* 2*a* 1， ①



*n* *n*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 所以当 *n* 1时， | | | | |  | *a*1=*S*1  2*a*1 1，解得 *a*1 1，················································ 1 分 | | | |
|  | 当 *n* 2 时， | | |  | *S* 1 2*a* 1 1  ， ②······························································ 2 分  *n* *n* | | | | |
|  | ① ②，得 |  | *a* 2*a* 2*a* ，即  *n* *n* *n*1 | | | | |  | ， 所以 *a* 2*n*1 . ································ 3 分  *a* 2*a*  *n* *n*1 *n* | | |

设数列

*b* 的公差为 *d* ，

*n*

由数列

*b* 的前三项和为 3，得3*b* 3*d* 3 ，即*b* *d* ，······························ 4 分

*n* 1

1 1

则

*b*3  *b*1  2*d* 1 *d*,*b*5  *b*1  4*d* 1 3*d* ，····················································· 5 分

又因为 2

*b* *b* *b* ，所以 (1 *d*)2 1 3*d* ，

3 2 5

解得 *d* 1或 *d* 0 （舍去），所以*b*1 1 *d* 0，

所以*b* *n* 1．······················································································· 6 分

*n*

（Ⅱ）同解法一．

（18）本小题主要考查正弦定理、余弦定理、解三角形等基础知识，考查应用意识、抽象概括能力、运算

求解能力，考查分类与整合思想、函数与方程思想等．满分 12 分．

解法一：（Ⅰ）在△ *PBC* 中， *PB* 90 ， *PC* 30 3 ， *PCB* 120，

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 由正弦定理得， |  | *PB* *PC*    sin*PCB* sin*PBC* |  | ，即 |  | 90 30 3  sin120sin*PBC* |  | ， ·························· 3 分 |

1

解得sin*PBC* . ······················································································· 4 分

2

又因为在△ *PBC* 中， 0 *PBC* 60 ，所以 *PBC* 30， ································· 5 分

所以 *BPC* 30 ，从而 *BC* *PC* 30 3 ，

即 *B*,*C* 两个岛屿间的距离为30 3 n mile． ······················································· 6 分

（Ⅱ）因为 *ABC* 90 ， *PBC* 30 ，所以 *PBA* *ABC* *PBC* 9030 60，

在△ *PAB* 中， *PB* 90 ， *AB* 30 ，由余弦定理得，

2 2 2 2 1

*PA* *PB* *AB* 2*PB* *AB*cos60 90 30 29030 30 7 ． ···············10 分

2

根据“两点之间线段最短”可知，

最短航线是“ *P* *A* *B* *C* *P* ”或“ *P* *C* *B* *A* *P* ”，

其航程为 *S* *PA* *AB* *BC* *CP* 30 7 30 30 3 30 3 30 60 3 30 7 ．

所以应按航线“ *P* *A* *B* *C* *P* ”或“ *P* *C* *B* *A* *P* ”航行，

其航程为30 60 3 30 7 n mile． ·····························································12 分

解法二：（Ⅰ）在△ *PBC* 中， *PB* 90 ， *PC* 30 3 ， *PCB* 120，

由余弦定理得， *PB*2  *PC*2  *BC*2  2*PC* *BC*cos*PCB*，······································ 3 分

1

即902  (30 3)2  *BC*2  230 3 *BC* ( ) ，

2

解得 *BC* 30 3 或 *BC* 60 3 （舍去）．

即 *B*,*C* 两个岛屿间的距离为30 3 n mile． ······················································· 6 分

（Ⅱ）在△ *ABC* 中， *BC* 30 3 ， *AB* 30 ， *ABC* 90 ，

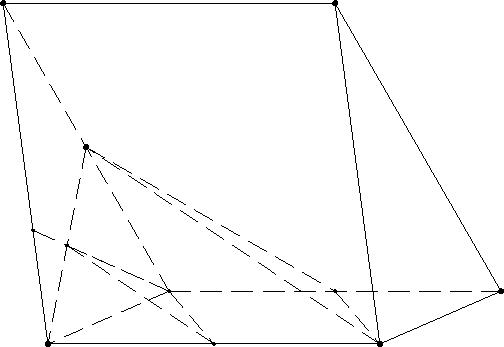
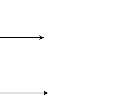
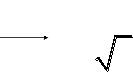
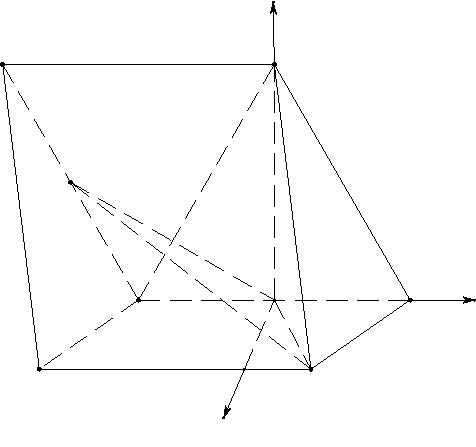
所以

2

*AC* *AB*2  *BC*2  302  30 3 60 ．·················································· 7 分

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*AB* 3



tan *ACB*

，因为 0 *ACB* 90，所以 *ACB* 30 ，

*BC* 3

在△ *PCA* 中， *PCA* *PCB* *ACB* 90 ， *PC* 30 3 ，

由勾股定理得， *PA* *PC*2  *AC*2  30 7 ．·····················································10 分

下同解法一.

（19）本小题主要考查直线与直线、直线与平面、平面与平面的位置关系及直线与平面所成的角等基础知

识，考查空间想象能力、推理论证能力、运算求解能力，考查化归与转化思想、数形结合思想、函

数与方程思想等．满分 12 分．

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 解法一：（Ⅰ）如图（1），在平面 | | | |  | *ABB* *A* 内，过点 *A* 作  1 1 |  | *AN*∥*B* *P* 交 *BB* 于点 *N* ，连结 *BQ* ，在△  1 1 |  | *BB* *Q*  1 | |
|  | | 中，作 |  | *NH*∥*BQ* 交 *BQ* 于点 *H* ，连结 *AH* 并延长交 *BC* 于点 *M* ，则 *AM* 为所求作直  1 | | | | | | |

线．··································································································· 2 分

***C*** ***C*1**

***Q***

***M*** ***H***

***A******A P***

**1**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | ***B*** |  | ***N*** |  | ***B*1** |

图（1）

·················································································································· 4 分

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | （Ⅱ）连结 | |  | *PC* ，  1 | | | | |  | *AC* ，∵  1 |  | *AA*1  *AC* *A*1*C*1  4,*C*1*A*1*A* 60 ，∴△ | | | | | | | | | |  | *AC* *A* 为正三角形.  1 1 | |
|  | | ∵ *P* 为 | | |  | *AA* 的中点，∴  1 | | | | | | |  | *PC* *AA* ，  1 1 | | |
|  | | 又∵侧面 | | | | |  | *ACC* *A* 侧面  1 1 | | | | | | |  | *ABB* *A* ，且面  1 1 | |  | *ACC* *A* 面  1 1 |  | *ABB* *A* *AA* ，  1 1= 1 | | |

*PC* 平面 *PC* 平面 *ABB* *A* . ···················5 分

*ACC* *A* ，∴

*z*

1 1 1 1 1 1

***C*** ***C*1**

在平面 *ABB* *A* 内过点 *P* 作 *PR* *AA* 交 *BB* 于点 *R* ，

1 1 1 1

|  |  |  |  |
| --- | --- | --- | --- |
|  | 分别以 |  | *PR*,*PA* ,*PC* 的方向为 *x* 轴，*y* 轴，*z* 轴的正  1 1 |

方向，建立如图（2）所示的空间直角坐标系 *P* *xyz* ，

***Q***

|  |  |  |  |
| --- | --- | --- | --- |
|  | 则  *P*(0,0,0), *A* (0,2,0), *A*(0,2,0),*C*(0,4,2 3),  1  *C*1(0,0,2 3) . |  | ***P***  ***A*** ***A*1** |

*y*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ∵*Q* 为 *AC* 的中点，∴点*Q* 的坐标为 (0,3, 3), | | | | | | | | | | | | | | | | |  | ***B*** |  | 图（2） |  | *x* |  | ***R*** |  | ***B*1** |
|  | | ∴ |  | *A*1*C*1  (0,2,2 3),*PQ* (0,3, 3). | | | | | | | | |
|  | | ∵ |  | *A*1*B*1  *AB* 2,*B*1*A*1*A* 60,∴ | | | | | |  | | *B*1( 3,1,0),∴  *PB*1  ( 3,1,0) .······················ 7 分 | | | | | | | | | | | | | | | | | | |
|  | | 设平面 | | |  | *PQB* 的法向量为 ***m*** (*x*, *y*,*z*) ，  1 | | | | | | | | | |
|  | | 由 |  | 0,  *PQ* ***m*** 3*y* 3*z* 0,    得  *PB* ***m***  0 3*x* *y* 0,    1 | | | | |  | | ··························································· 9 分 | | | | | | | | | | | | | | | | | | |
|  | | 令 *x* 1，得 *y* 3, *z* 3,所以平面 | | | | | | | | | | |  | | *PQB* 的一个法向量为 ***m*** (1, 3,3) .···10 分  1 | | | | | | | | | | | | | |
|  | | 设直线 | | |  | *AC* 与平面  1 1 |  | *PQB* 所成角为 ，  1 | | | | | | | | |

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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | | *AC* ***m***  则sin | cos *AC* ,***m*** | 1 1    1 1  | *AC* || ***m*** |  1 1 | | | | | | | | | | |  | 39  13 |  | ， | | |
|  | | | | 即直线 |  | *AC* 与平面  1 1 | | | | |  | *PQB* 所成角的正弦值为  1 | | | | | | |  | 39  13 | | | |  | ．···································12 分 | | | | |
| 解法二：（Ⅰ）如图（3），在平面 | | | | | | |  | *ABB* *A* 内，连结  1 1 | | | | |  | *A* *B* ，过点 *A* 作  1 | | | | | | | |  | *AN*∥*B* *P* 交  1 | | |  | *A* *B* 于点 *N* ，连结  1 |  | *AC* ，  1 | | |
|  | 在 |  | *A* *BC* 中，过点 *N* 作  1 | | | | | |  | *NM*∥*AC* 交 *BC* 于点 *M* ，连结 *AM* ，则 *AM* 为所求作直线． 2 分  1 | | | | | | | | | | | | | | | | | | | | |

***C*** ***C*1**

***Q***

***M***

***A P***

***A*1**

***N***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | ***B*** |  | 图（3） |  | ***B*1** |

·················································································································· 4 分

***C*** ***C*1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | （Ⅱ）如图（4），连结 | | |  | *AB* ，∵  1 |  | *AC* // *AC* ，  1 1 |
|  | ∴直线  *AC* 与平面 | *AC* 与平面  1 1  *PQB* . ································· 5 所成角 分  1 | *PQB* 所成角等于  1 | | | | | | | | | |  | ***Q*** |
|  | ∵   |  | | --- | | *AA*1  *AC* 4,*C*1*A*1*A* 60,  1 ,  *AA* *AC* 的中点， | |  |   *P*,*Q* 分别为 | | | | | | | | | |  | ***S*** ***P*** ***T***  ***A******A***  **1** | | | | | | | | |
|  | ∴在△ *APQ* 中， *AP* *AQ* 2,*PAQ* 120, | | | | | | | |  | ***B*** | | | | | |  | 图（4） |  | ***B*1** |

由余弦定理得 *PQ*2  *AP*2  *AQ*2  2*AP* *AQ*cos*PAQ*，解得 *PQ* 2 3 .······················ 6 分

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 连结 |  | *AC* ，则△*AC* *A* 为正三角形，∴*C* *P* *AA* .  1 1 1 1 1 | | | | | | | | | | | | | | | | | | |
|  | 又∵平面 | | | | |  | *ACC* *A* 平面  1 1 | | | | | |  | *ABB* *A* ，且平面  1 1 | | | | |  | *ACC* *A* 平面  1 1 | | |  | *ABB* *A* *AA* ，  1 1= 1 |  | *C* *P* 平面  1 | | | | | | |  | *ACC* *A* ，  1 1 | | | |
|  | ∴  *C* *P* 平面  1 | | | | | | |  | *ABB* *A* .  1 1 |
|  | 在平面 | | |  | *ACC* *A* 内，过点 *Q* 作*QS* //*C* *P* 交  1 1 1 | | | | | | | | | | | |  | *A* *A* 延长线于点 *S* ，则*QS* 平面  1 | | | | | | | | | | |  | *ABB* *A* . ······ 7 分  1 1 | | | | | | |
|  | 连结 *SB* ，∵  *SB* 平面  1 1 | | | | | | | | | |  | *ABB* *A* ,∴*QS* *SB* .  1 1 1 | | | | | | | | |
|  | 1 1  *QS* *C* *P* *AC* sin*C* *A* *A* 3 ,  又∵  1 1 1 1 1  2 2 | | | | | | | | | | | | | |  | *SA* *AQ*cos*CAS* *AQ*cos*C* *A* *A* 1, ∴  1 1 | | | | | | | | | | | | | | |  | *A* *S* .  1 5 | | | |
|  | 在△*A* *SB* 中，由余弦定理得 *SB*12  *A*1*S*2  *A*1*B*12  2*A*1*S* *A*1*B*1 cos*B*1*A*1*A*,解得  1 1 | | | | | | | | | | | | | | | | | | | | | | | | | |  | *SB*  1 19. | | | | | | |

∴ 1 2 12 22.

*QB* *QS* *SB* ··········································································· 8 分

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 又∵在△*A* *PB* 中，  1 1 |  | 1  *B* *A* *P* 60, *A* *B* *AB* 2, *A* *P* *AA* 2 ，  1 1 1 1 1 1  2 | | | |
|  | ∴△*A* *PB* 为正三角形，∴  1 1 | | |  | *PB* .  1 2 |

*PQ* *PB* *QB* 3

2  2  2 13

在△*PQB* 中， cos*QPB* , *QPB*

∴

1 1 sin ，

1

2*PQ* *PB* 4 4

1  1

1

1 39

*S* *QP* *PB* sin*QPB*

∴ △ . ································································ 9 分

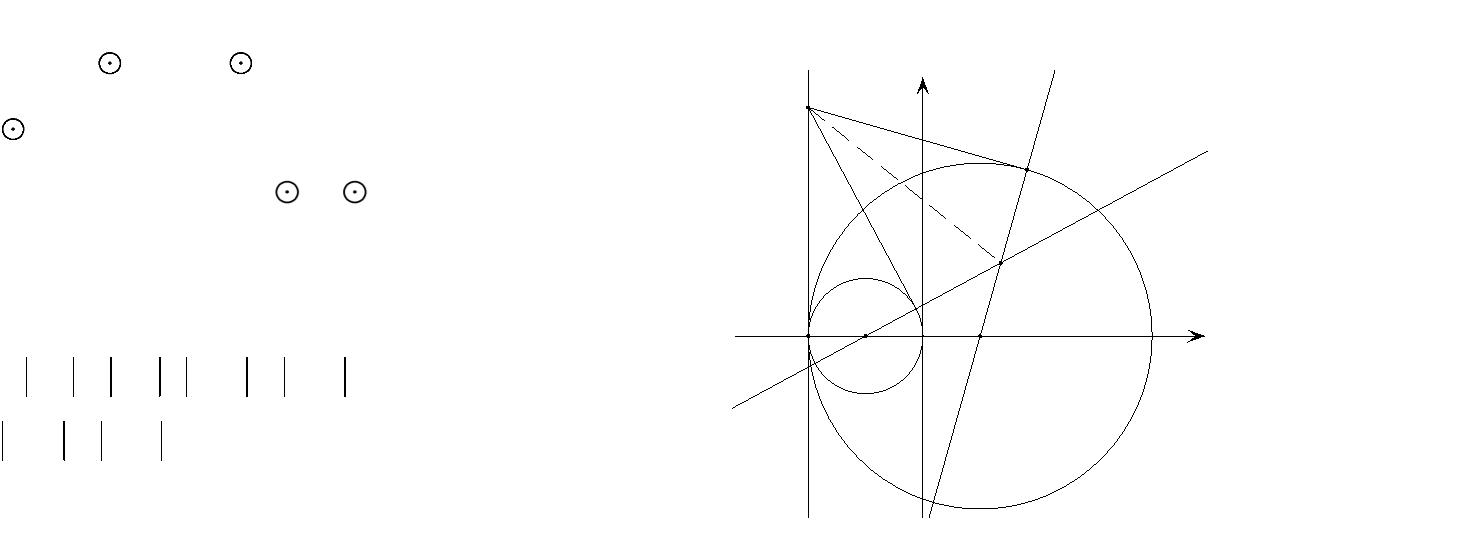
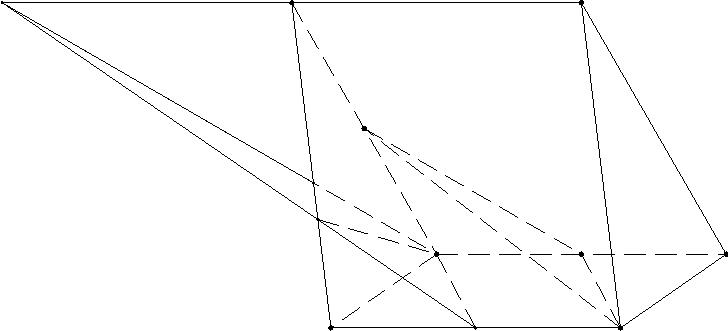
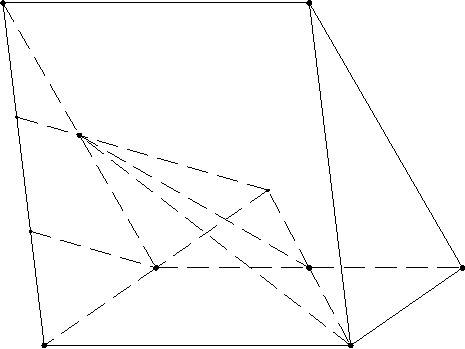
*PQB*1 1 1

2 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 取 |  | *A* *P* 的中点*T* ，连结  1 |  | *BT* ，则 1 1 , 1 3  *BT* *A* *P* *BT* ，  1 |

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1 1



∴ △ 3 ，*V* *S*△ *QS* 1. ············································10 分

*S* *AP* *BT*

*APB*1 1

*Q* *APB* *APB*

2 3

1 1

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | 设 *AC* 与平面 |  | *PQB* 所成角为 ， *A* 到平面 *PQB* 的距离为 *h* .  1 1 | | | | | | | | |
|  | | 1  ∴*V* *V* △ ，解得  ，即  1 *S* *h* 1  *A* *PQB* *Q* *APB* *PQB*  3  1 1 1 | | | | | | |  | 2 39  *h* . ·······································11 分  13 | | | | | | | | |
|  | | 2 39  *h* 39  13  ∴sin ，即直线  *AQ* 2 13 | | | | |  | *AC* 与平面  1 1 | | |  | *PQB* 所成角的正弦值为  1 | |  | 39  13 |  | ．·······12 分 | | |
|  | 解法三：（Ⅰ）如图（5），延长 | | | |  | *BA*,*B* *P* 交于点 *R* ，连结 *RQ* 并延长交 *BC* 于 *K* ，在△ *ABC* 中，过点  1 | | | | | | | | | | | |

*A* 作 *AM*∥*QK* 交 *BC* 于点 *M* ，则 *AM* 为所求作直线．··································· 2 分

***C*** ***C*1**

***K Q***

***R***

***M P***

***A***

***A*1**

***B***

***B*1**

图（5）

·················································································································· 4 分

（Ⅱ）同解法一．

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 解法四：（Ⅰ）如图（6），在平面 |  | *ABB* *A* 内，过点 *A* 作  1 1 |  | *AN*∥*B* *P* 交  1 |  | *BB* 于点 *N* ，在平面  1 |  | *AAC* *C* 内，  1 1 |

过点 *A* 作 *AS*∥*PQ* 交*C* *C* 的延长线于点 *S* ，连结 *SN* 交 *BC* 于点 *M* ，连结 *AM* ，则 *AM* 为所

1

求作直线. ····································································································· 2 分

***S*** ***C*** ***C*1**

***Q***

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ***M*** | |  | ***A*** | | |  | ***P*** | | | |  | ***A*1** |
|  | | ***B*** | | |  | ***N*** | | |  | ***B*1** |

·················································································································· 4 分

图（6）

（Ⅱ）同解法一.

（20）本小题主要考查圆的标准方程、椭圆的标准方程、直线与圆锥曲线的位置关系等基础知识，考查推

理论证能力、运算求解能力，考查数形结合思想、函数与方程思想、化归与转化思想等．满分 12 分．

解法一：（Ⅰ）因为 *C* 内切

*C* 于 *A* ，所以 *r* 1 2 ，解得 *r* 3 ，···································· 1 分

1 2

***y***

2 2

***P***

所以

*C* 的方程为

: *x* 1 *y* 9 .

2

***R***

|  |  |  |  |
| --- | --- | --- | --- |
|  | 因为直线 *PQ*,*PR*分别切 |  | *C*1, *C*2 于*Q*,*R* ， |
|  | 所以 1 , 2  *C* *Q* *PQ* *C* *R* *PR* .······························· 2 分 | | | |  | ***M*** |

连结 *PM* ，在 Rt△*PQM* 与 Rt△*PRM* 中，

***Q***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *PQ* *PA* *PR* , *PM* *PM* , ························· 3 分 |  | ***A*** ***C*1 *O*** ***C***  **2** |  | ***x*** |

所以 *QM* *RM* ，

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